



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

$86164 \div 3579 = 24.075$  times as fast as at present.

$\therefore$  One revolution per hour.

By this method we can find the time of revolution for any latitude. It is

$$t=4 \sqrt{\frac{3b}{L'}} \left\{ \frac{\left(1 - \frac{1/\sqrt{(1-e^2)} \sin \theta}{1/\sqrt{(1-e^2) \sin^2 \theta}}\right)^{\frac{1}{2}}}{\left(1 + \frac{1}{4}e^2 \sin^2 \theta\right)} \right\}$$

where  $b$  is taken in feet, and  $L'$  in inches.

The following table gives gravity and the length of the second's pendulum for every five degrees under existing conditions.

DEGREES	GRAVITY		SECOND'S PENDULUM	
	POUNDALS	DYNES	INCHES	mm
0°	32.0002682	978.09	39.017088	991.01
5°	32.0915197	978.13	39.018610	991.05
10°	32.0952903	978.25	39.023194	991.17
15°	32.1014356	978.43	39.030666	991.36
20°	32.1097470	978.69	39.040771	991.62
25°	32.1200159	979.00	39.053257	991.93
30°	32.1319053	979.36	39.067713	992.30
35°	32.1450623	979.76	39.083710	992.71
40°	32.1590697	980.19	39.100741	993.14
45°	32.1735424	980.63	39.118337	993.59
50°	32.1880023	981.07	39.135918	994.03
55°	32.2020226	981.50	39.152965	994.47
60°	32.2151796	981.90	39.168962	994.87
65°	32.2270722	982.26	39.183422	995.24
70°	32.2373379	982.57	39.195903	995.56
75°	32.2456621	982.83	39.206024	995.81
80°	32.2517955	983.02	39.213482	996.00
85°	32.2555651	983.13	39.218065	996.12
90°	32.2568167	983.17	39.219587	996.16

## A PROBLEM CONNECTED WITH MERSENNE'S NUMBERS.

By HARRY S. VANDIVER, Bala, Pa.

In Sir W. R. Ball's *Recreations and Problems* (London, 1809), page 33, I find the following:

"A curious proposition which comes from China, and which I believe appears here in print for the first time, is that  $(2^n - 2)/n$  is an integer if  $n$  is a prime and is not an integer if  $n$  is not a prime. The first of these statements is at once demonstrable; but I have not succeeded in proving the second part of the propo-

sition, which seems to introduce considerations similar to those involved in the theory of Mersenne's Numbers."

This proposition is of extraordinary importance, since, if we suppose it true then it gives a complete analytic definition of a prime. For instance, to find whether or not  $a$  is a prime it would only be necessary to calculate the residue of  $2^{a-1}$  with respect to  $a$ . If we find  $2^{a-1} \equiv 1 \pmod{a}$  then  $a$  is a prime, but if  $2^{a-1}$  is not congruent to 1  $\pmod{a}$  then  $a$  is composite.

The object of the following investigation is to show that the second part of the proposition is *false*.

To prove this falsity it is sufficient to find a value  $n$ —an odd composite such that  $2^{n-1} \equiv 1 \pmod{n}$ .....(1).

Let us suppose, first that  $n$  is the simplest form of odd composite  $= p \times q$ , where  $p$  and  $q$  are primes. Then, since  $\phi(n) = (p-1)(q-1)$ , we have

$$2^{(p-1)(q-1)} \equiv 1 \pmod{pq}$$

by Fermat's Generalized Theorem. If we assume that  $m$  is the smallest number such that  $2^m \equiv 1 \pmod{pq}$ , then  $m$  must be a divisor of  $\phi(n)$  and if (1) is possible then  $n$  must be a multiple of  $m$ . (Serret's *Algebra*, Sup. Vol. 2, page 48).

These conditions may be written

$$\begin{aligned} pq - 1 &\equiv 0 \pmod{m} \\ (p-1)(q-1) &\equiv 0 \pmod{m} \end{aligned}$$

Comparing these congruences, we find

$$p \equiv 1 \pmod{m}, \quad q \equiv 1 \pmod{m}.$$

Then to prove the possibility of (1) it is sufficient to find values,  $m, p, q$ , such that  $2^m \equiv 1 \pmod{pq}$  where  $q \equiv 1 \pmod{m} \equiv q$  and  $m$  is not greater than  $\phi(pq)$ .

To find these values, assign to  $m$  small integral values in succession to find, by trial, numbers for  $p$  and  $q$  corresponding.

If  $m=1$  to 9 inclusive, no appropriate values can be found for  $p$  and  $q$ , but if  $m=10$ , then we can put

$$p = 10 \times 1 + 1 \text{ and } q = 3 \times 10 + 1,$$

$$\text{and then} \quad 2^{10} \equiv 1 \pmod{31 \times 11}$$

$$\text{and therefore} \quad 2^{340} \equiv 1 \pmod{31 \times 11}$$

and the second part of the proposition originally quoted is thus seen to be false.

If  $m=11$ , then putting  $p=23$ ,  $q=89$ ,

$$2^{2046} \equiv 1 \pmod{2047}.$$

In the same manner we can find other values of  $m$ ,  $p$ , and  $q$ , and the number of possible sets does not appear to be limited.

The first part of the proposition, namely,  $(2^n - 2)/n$  is an integer when  $n$  is prime, is but a particular case of Fermat's Theorem that  $a^{p-1} \equiv 1 \pmod{p}$  when  $p$  is prime and  $a$  is prime to  $p$ .

*Bala, Pa., Feb. 1, 1902.*

---

## GEOMETRIC DERIVATION OF CERTAIN TRIGONOMETRIC FORMULAE.

---

By PROFESSOR L. E. DICKSON.

---

Students of trigonometry find it interesting to have, in addition to the usual proof, the following geometric derivation of the formulae used in the solution of a plane triangle of given sides. The only trigonometry used is the *definition* of the tangent ratio.

The first step is the usual geometric proof (by means of the theorem giving, in terms of the sides, the projection of one side on another side) of Heron's formula :

$$\Delta = \sqrt{[s(s-a)(s-b)(s+c)]}.$$

The next step is the evaluation of the radius  $r$  of the circle inscribed in the triangle. Its center is  $O$ , the areas of the triangles  $AOB$ ,  $BOC$ ,  $COA$  are  $\frac{1}{2}cr$ ,  $\frac{1}{2}ar$ ,  $\frac{1}{2}br$ , respectively. Hence

$$\Delta = \frac{1}{2}(a+b+c)r = sr.$$

$$\therefore r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

The next step is the simple proof that the length of the tangent from  $A$  to the inscribed circle is  $AE = AD = s - a$ ; from  $B$ ,  $BE = BF = s - b$ , and from  $C$ ,  $CF = CD = s - c$ . Then

$$\tan \frac{1}{2}A = \frac{r}{s-a}, \quad \tan \frac{1}{2}B = \frac{r}{s-b}, \quad \tan \frac{1}{2}C = \frac{r}{s-c}.$$

These are the most convenient formulae for the solution of a triangle of given sides. We may, however, derive at once the formulae for  $\tan \frac{1}{2}A$ ,  $\sin \frac{1}{2}A$ ,  $\cos \frac{1}{2}A$  in terms of  $a$ ,  $b$ ,  $c$  only. Evidently,

